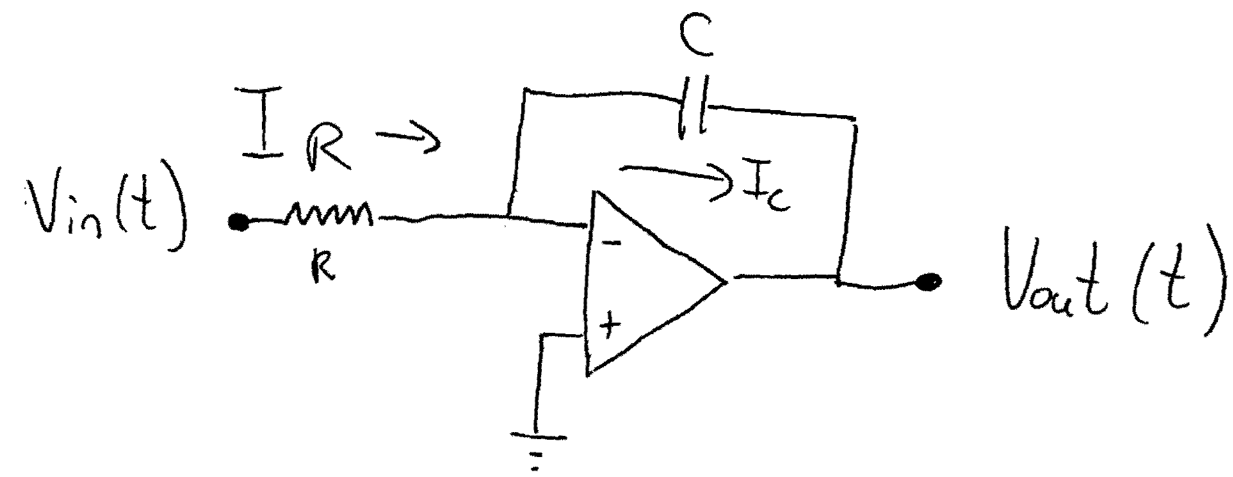
Op-Amp Filters:

Integrating (Low Pass) Filter:



As with most inverting configurations use conservation of current.

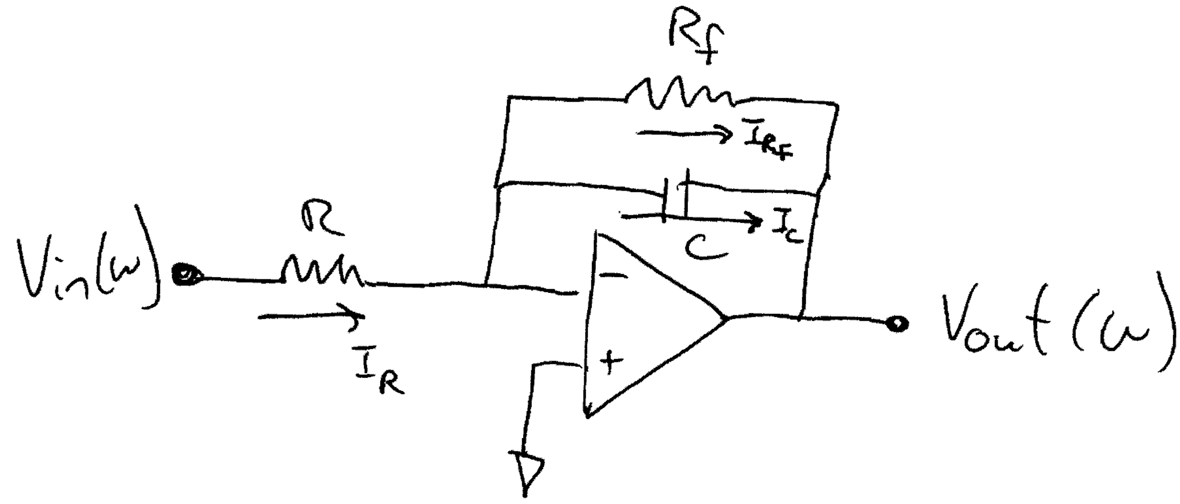
$$I_R = \frac{V_{in} - V_-}{R} \quad ; \quad I_c = \frac{dQ_c}{dt} = C\frac{dV_c}{dt}$$

Use $V_- = 0$

$$I_c = C\frac{d}{dt}(V_- - V_{out})$$

$$\frac{V_{in}(t)}{R} = -C\frac{d}{dt}V_{out}(t)$$

$$\Rightarrow \boxed{V_{out}(t) = -\frac{1}{RC}\int_0^t V_{in}(t')\,dt'}$$



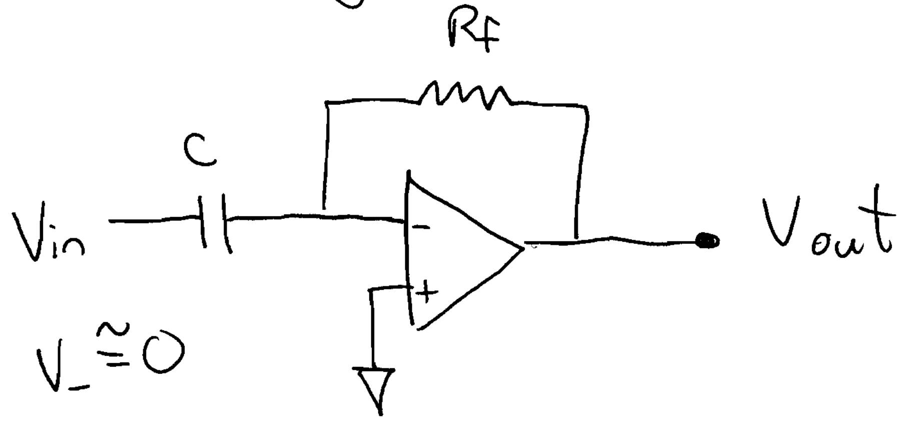
$$I_R = \frac{V_{in}}{R} \; ; \quad I_C = \frac{V_C}{Z_C} = -i\omega C\, V_{out}(\omega)$$

$$I_{Rf} = -\frac{V_{out}}{R_f} \; ; \quad I_R = I_C + I_{Rf}$$

$$\frac{V_{in}}{R} = -V_{out}\left(i\omega C + \frac{1}{R_f}\right)$$

$$\boxed{V_{out}(\omega) = -\frac{R_f}{R} \cdot \left(\frac{1}{i\omega C R_f + 1}\right) V_{in}(\omega)}$$

Differentiating (High Pass Filter) Filter: ③



$V_- \cong 0$

time domain: $V_{in} = V_{in}(t)$, $V_{out} = V_{out}(t)$

$I_c = C\dfrac{dV_{in}}{dt}$; $I_{Rf} = \dfrac{-V_{out}}{R_f}$; $I_c = I_{RF}$

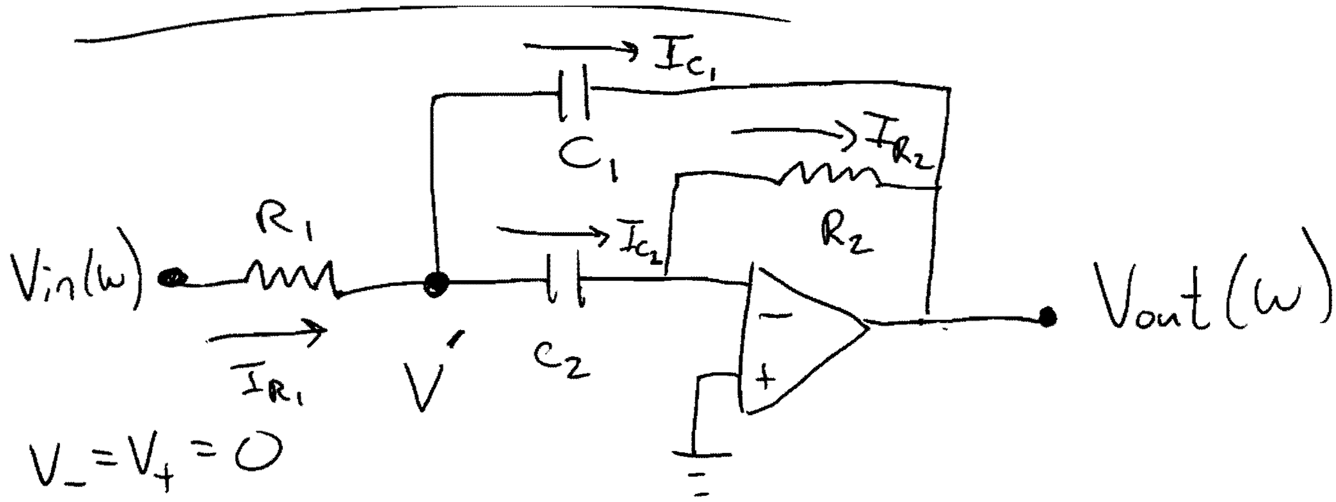
$$\Rightarrow \boxed{V_{out}(t) = -R_f C \frac{dV_{in}(t)}{dt}}$$

frequency domain: $V_{in} = V_{in}(\omega)$
$V_{out} = V_{out}(\omega)$

$I_c = \dfrac{V_c}{Z_c} = -i\omega C V_{in}(\omega)$; $I_{Rf} = -\dfrac{V_{out}(\omega)}{R_F}$

$$\Rightarrow \boxed{V_{out}(\omega) = -i\omega C R_f V_{in}(\omega)}$$

Band Pass Filter :



$V_- = V_+ = 0$

$$I_{R_1} = \frac{V_{in} - V'}{R_1} \; ; \quad I_{C_2} = -i\omega C_2 V'$$

$$I_{R_2} = \frac{-V_{out}}{R_2} \; ; \quad I_{C_1} = -(V' - V_{out})\, i\omega C_1$$

(1) $I_{R_1} = I_{C_1} + I_{C_2}$

(2) $I_{C_2} = I_{R_2}$

Use two equations to remove V' variable and solve for V_{out}.

$$\boxed{V_{out}(\omega) = \frac{(-i\omega C_2 R_2)\, V_{in}(\omega)}{i\omega R_1 (C_1 + C_2) + C_1 C_2 R_1 R_2 \omega^2 - 1}}$$